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FINAL SCIENTIFIC REPORT

RESEARCH ON ELEMENTARY PARTICLES

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Professor Abdus Salam and Professor P.T. Matthews.  
Department of Physics,  
Imperial College of Science & Technology,  
LONDON - ENGLAND.



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ABSTRACT

The research described in this report extends over a broad range of topics in the theory of elementary particles and related fields, especially concerned with space-time and internal symmetries, current algebras and Regge-pole theory.

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1. INTRODUCTION.

Our group has devoted its attention to a large number of research projects covering a wide variety of subjects within the general field of elementary particle theory. Among the most important of these have been studies of group-theoretical aspects of space-time and internal symmetries of elementary particles. Particularly significant results have been obtained in the theory of symmetries of scattering amplitudes at small momentum transfer, and in Regge-pole theory. Studies of various processes have been undertaken using the techniques of current algebras and finite-energy sum rules. Other fields in which significant advances have been made are quantum optics, astrophysics and axiomatic quantum field theory. These researches are summarized below and described in detail in the publications listed at the end of the report.

2. SPACE-TIME SYMMETRIES.

Scattering amplitudes can be decomposed in various ways, appropriate to different ranges of energy and momentum transfer. These decompositions include the partial-wave expansion, useful at low energies, and the expansion in terms of Regge poles, which is convenient at high energies and small momentum transfers. In the special case of zero momentum transfer, there is an extended symmetry which imposes constraints on the expansion parameters, and a corresponding special decomposition, related to limiting forms of the more general expansions. A comprehensive group-theoretic discussion of these decompositions has been given which is fully relativistically covariant and which clearly exhibits the relationships between the various special cases. This treatment also yields very convenient techniques for handling the practical problems involved in studies of many processes.

The relationships between helicity amplitudes and invariant amplitudes have also been studied extensively. A general discussion of this problem has been given, which in particular yields very simply the set of invariant amplitudes free of kinematic singularities for any given process. This method is well adapted to the determination of the constraints imposed by the symmetry at zero momentum transfer. It has been extended to cover not only massive particles but also particles of zero mass, such as the photon for which special problems associated with gauge invariance arise.

### 3. INTERNAL SYMMETRIES AND CURRENT ALGEBRAS.

A variety of mathematical problems involved in the study of internal symmetry groups, especially of the groups  $U(6,6)$  and  $SL(6,C)$  have been investigated. In particular, important results have been obtained in the representation theory of these groups, and of  $SO(n)$ ,  $SU(n)$  and  $SU(n) \times SU(n)$ .

The precise physical significance of the assumptions involved in internal symmetry theories, and the correct formulation of such theories has been another subject of several investigations. One of the most promising techniques is that of current algebras, in which the basic content of the theory is expressed in the assumption that the vector and axial vector currents of the theory form a representation of an appropriate algebra. The possibility of saturating the sum rules derived from current algebras by a small number of resonant states has been considered, and the results compared with experiment.

An alternative model which can also incorporate the internal symmetry in a rather different way is the absorption model. Here the basic assumption is that in peripheral scattering processes the coupling constants

which describe the coupling of the exchanged particle have the symmetry described by, for example, the group  $U(6,6)$ . The insertion of these constraints into the absorption model yields powerful predictions which have been successfully compared with experiment for a variety of scattering processes.

#### 4. REGGE-POLE THEORY AND FINITE-ENERGY SUM RULES.

The theory of Regge poles, already discussed above, has also been investigated in an approximation scheme based on the ladder approximation. This technique yields the possibility of computing the parameters of a Regge trajectory - the trajectory function and residue functions - in a self-consistent manner analogous to a bootstrap theory. The results obtained so far are extremely promising, and full computer calculations are now under way.

Regge-pole theory is also involved in another subject, the study of finite-energy sum rules. The use of the asymptotic behaviour derived from Regge-pole theory permits the derivation of sum rules which extend only to a finite upper limit of energy rather than to infinity, and which are therefore more amenable to direct comparison with experiment. The incorporation of internal symmetry into this scheme leads to interesting predictions which have been compared with experiment.

For certain amplitudes, Regge asymptotic behaviour requires so-called superconvergent sum rules, which are in effect restrictions imposed on scattering amplitudes by the assumptions of Regge theory. Here too the incorporation of  $U(6,6)$  symmetry has led to remarkable predictions. An extensive study of these relations has been made, especially for the nonet of  $2^+$  mesons. The results are generally in good agreement with experiment, in those cases where the

experimental data are adequate to make a meaningful comparison.

#### 5. WEAK INTERACTIONS.

Applications of current algebra to weak interactions have been studied, and calculations made of various processes, for comparison with experiment. A study has also been made of the  $\Delta I = \frac{1}{2}$  rule in non-leptonic weak interactions. The experimental status of the rule has been examined and a theory proposed which fits the available data. A possible explanation of the violation of CP symmetry has also been proposed.

#### 6. AXIOMATIC FIELD THEORY.

Several nonrelativistic model field theories which are soluble, or can be shown to have well-behaved solutions, have been studied. In particular a large class of model theories which exhibit spontaneous symmetry breaking have been investigated, using the rigorous mathematical techniques of axiomatic quantum field theory. Certain possible generalizations of the Heisenberg theory of the ferromagnet have been investigated, and it has been demonstrated that a ferromagnetic continuum is impossible.

Axiomatic studies of representations of current algebras have also been made, with very interesting results. One result of importance is a 'no-go theorem' which demonstrates the impossibility of constructing representations with certain properties which had commonly been assumed in the literature.

#### 7. QUANTUM OPTICS.

Interactions with matter of intense electromagnetic fields such as those produced by a laser, have been studied. In particular, a quantum-mechanical calculation has been made of stimulated Raman scattering, a phenomenon of considerable recent experimental interest. An investigation

has also been made of certain molecular features which appear in the spectra of atoms trapped in inert gas matrices.

A related topic which has been the subject of lengthy studies is that of the infrared divergences of quantum electrodynamics. A formalism which uses generalized coherent states to describe the emitted infrared photons has been developed, which is capable of reproducing in a more physically satisfactory way the conventional results, and moreover which can be used in circumstances where the conventional theory is inadequate.

#### 8. ASTROPHYSICS.

An interesting proposal has been made that in certain circumstances the velocity of sound in ultradense matter may exceed the velocity of light. This proposal would remove certain difficulties in the way of one of the theories of quasars. Some simple models which exhibit this behaviour have been developed, and its various implications investigated.

A study has also been made of the role of certain elementary-particle interactions, beyond the scope those hitherto generally considered, in astrophysical problems. Another important topic considered was crystallization and torsional oscillations of superdense stars.

#### 9. CONCLUSIONS.

In addition to the topics mentioned in the preceding sections, several other subjects have been investigated. Notable among these were a study of the Lovelace equation, using partial-wave analysis, a consideration of certain aspects of parastatistics, and several phenomenological studies of various reaction processes.

Our group has continued to make a very significant contribution to the advancement of our understanding of the basic laws of physics. Very considerable progress has been made, particularly by using the techniques of current algebra and Regge-pole theory. The symmetry properties of the elementary particles are beginning to take on a more established and secure appearance. Although we still lack a comprehensive theory, certain portions of the theory which must eventually emerge have become much clearer. Much work however remains for this and other groups.

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